THE OUTLOOK WITH REGARD TO SCHOOL MATHEMATICS.[1]

Few teachers, probably, realize that they are living in a period of great mathematical development. New phases of old problems, new problems and new methods are continually demanding some new modification in mathematics or else out and out new methods in mathematical procedure.

Critical studies in physics and astronomy have in recent years called forth a new theory of relativity which is essentially mathematical. No one but the mathematician could compass this problem. The result is a serious revision of long standing theories.

Much attention has been given lately to the study and discovery of empirical formulas for the concise statement of physical, economic or social phenomena. Recently several treatises have been published in this field. Only a few weeks ago the writer was called upon to assist in the determination of a formula for simplifying the cost estimates on contracts for iron castings. One of the prominent engineers of the city was working out the problem.

Just the other day a professional educator came for advice in regard to a formula which he had arrived at empirically. He wished to eliminate the element of probability from the scores of pupils in a certain test. And so it goes on. It may be of interest to add that the mathematical side of both problems involved the use of certain parts of the traditional courses in algebra.

The recent income tax legislation has called forth new interest in statistical methods in several great industries to such an extent that private schools of statistics designed especially for the purpose have sprung up. The writer had a conversation with the principal of one of these schools a few days ago. Here again the special course is built on a modicum of standard mathematics and general statistical method. Much use is made

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of graphic representation and the use of calculating tables, both of which are begun in any well-designed course in algebra.

These examples are significant. They point not to the death of mathematics but to a changed emphasis and a modification in the content of school courses. Some topics must be further pared down to make room for the new problems and their attendant theory. Nothing vital and universal will be nor can safely be given up. Typical mathematical procedure will still be essential. Instead of an even emphasis on all the pages of an antiquated text, as the writer has observed in some instances, there must be a more critical selection of the universal fundamentals for greater emphasis and a new set of applied problems suited to present progress. This does not mean the elimination of all old problems but a closer selection. The time spent in first-year algebra factoring such expressions as $x^{16} - y^{16}$ or $a^{21} - b^{21}$ can be more profitably spent in further drill on simpler cases and in applications. It means more attention to certain graphical methods, especially to the interpretation of graphs in connection with statistical data and simple forms of functions. Some study of averages, such as arithmetical mean and weighted mean are easily within the grasp of high-school pupils. In the more advanced classes the idea of chance and probability can be given with advantage. Probability enters a large number of scientific and social studies. As an illustration, consider a test questionnaire of twenty-five questions, each with six suggested answers, only one of which is correct. Ask the candidate to mark the correct answers. By the law of chance he will get one sixth correct if he knows absolutely nothing. One might stand off and shoot at the questionnaire with a shotgun and be sure that one sixth the answers hit by shot will be correct. It is sometimes desirable to eliminate this chance element from the test. This is a mathematical problem.

Some General Observations.

1. The tendency of all organized society is toward more scientific study of relationship in economic, social and natural phenomena.

2. Scientific study invariably tends to measurement and functionality between elements or parts of observed phenomena.

3. Measurement and function study are preeminently the field of mathematical study.

4. Some knowledge of mathematics is desirable and necessary to the equipment of any student of natural and social phenomena.

5. The mathematical knowledge necessary for the above purposes must include some scientific training in typical mathematical methods and some skill in mathematical technique.

6. There must be a certain ability to apply the mathematical methods most adapted to any given particular situation.

7. The rapid changes in social and economic affairs in the last few years have opened fields of study to such a degree as not found in any similar period in history wherein mathematical training is needed. This means a most unusual opportunity for teachers and students of mathematics.

8. It probably is not mathematics that has been criticized but rather the too conservative disciples of the science in their administration of the content and methods of elementary courses. The process of adaptation has been too slow.

9. The mere lesson hearer and the text book slave must be eliminated or rejuvenated. Teachers of mathematics must have a contact with affairs, not shop problems alone, if our subject is to perform its natural and possible service to civilization.

We are in the throes of a spasm of educational measurements. We are attempting to measure almost every thing in school. This is only a chapter in the mathematization of all science. Often we have a desire to measure and do not know what or how to measure. This desire may be a sign of good intentions and scientific spirit. In attempting to measure we may learn what to measure and how to measure it. We are indeed making some progress in measuring the more obvious attainments in mathematics, viz., formal technique. The tests in algebra measure with some certainty the pupil's attainment in formal manipulations. Technique is essential to any use of mathematical applications in science. But technique alone will generally not be sufficient to ensure the use of mathematical methods in practical situations. We are therefore far from measuring the whole of the results of good teaching or

the ultimate values of mathematics as an educational instrument in a complicated and scientific civilization. We have not gone far enough to know definitely just what to teach and why and how. We should not however be discouraged for the problem is a large one and cannot be solved in a day or in a year. It is a problem for a generation. We can and must contribute our part toward the solution and our children will see it finished.

An able psychologist and mathematician has laid down some definite general aims in teaching mathematics:

1. Elementary manipulations.
2. Analysis of a problem or situation.
3. Symbolization of data and relations.
4. Manipulations necessary to a solution.
5. Interpretation of results.
6. Power to extend the methods to new fields.

We should as soon as possible take steps to measure accomplishment under these heads. It is conceivable that two or more of these may be tested by the same questionnaire. It may be necessary to further subdivide the heads to make satisfactory measurements. We must first determine what distinct general steps are involved in the solution of a problem. It is probable these tests will be more difficult to formulate than the technique tests.

Tests made during the progress of a course are no doubt valuable, but there should be general and broad tests at longer intervals covering the entire work of two or more years. Some of the richest generalizations and transfer values do not come until the end of an extended term of study or even after the subject has been laid aside.

The psychology of specific attainment has held the stage for some time. During this time constructive scientific psychology has been "sawing wood." There has come a doctrine of generalized habits. The attacks of the destructive psychologists have been halted and we are entering an era of better considered principles which promise much for the theory of education. The writer has stated in another paper: "Psychology cannot teach us how to teach but can and must furnish the basis on which we are to build the theory and practice

of teaching." The doctrines of generalization and of general habits seem destined to play an important rôle in educational theory during years to follow. Our tests and measurements from now on must recognize these results of psychological research. Those who have "buried formal discipline" will not necessarily be qualified to lead us in the next steps.

Mathematics will not be entirely nor even almost eliminated from the curriculum. But a new type of mathematics which is better adapted ultimately to our recent developments will take the place of the traditional courses in the secondary schools. Instead of the mature and finished treatises of Euclid in geometry and a similarly complete algebra we shall select more carefully the essentials of theory and technique and turn to newer applications and to different estimates of value. The geometry of Euclid is now only a small part of the available material in geometry for school purposes. The traditional algebra is only a part of the available analytic material. We shall continue the pruning process on one hand while on the other we shall graft new scions into the old trunk. The species of the fruit may not be different but the variety and the flavor will be different. Less elaborate formal exercises and more graphic representation, some treatment of averages, theory of investment, etc., will be included.

In geometry many complicated and little used theorems may well be omitted to make room for drawing to scale, new applied problems and some modern developments in geometry.

The new course in mathematics will be more of a composite, broader and cast in closer touch with the progress of civilization. This modification of course refers to the school courses in mathematics. The colleges and universities can never dispense with abstract mathematics. For here are formed the foundations of scientific advancement and research. Certain groups of students may with advantage be given more concrete courses.

All these changes must be made while still retaining the typical fundamental truths and methods of mathematics. There must remain enough of theory, rationalized and logically presented, in order to secure the power which must come and can come only from mathematical study. The course must retain such distinctive characteristics as to make the subject

self-perpetuating and universal in human thought. To reduce mathematical instruction to the incidental use of formulas in engineering, chemistry and accounting would be a serious error in educational policy.

One writer has recently stated that "Before 1700 there were six important scientific discoveries or inventions. Since 1800 there have been fourteen: The first six are writing, numeral system, compass, printing, telescope and barometer. The orthers are steam engine, railways, steam navigation, telegraph, telephone, friction matches, gas lighting, anæsthetics, antiseptics, electric lighting, phonograph. In current times we may add the cinematograph, wireless telegraphy and telephony, electric navigation, internal combustion engines, aeronautics, steel production, submarine navigation.

It is not too much to say that without mathematically formulated sciences our civilization would never have been possible. If the study of mathematics both high and low were to be stopped now, a relatively short period would see the whole civilized social and economic structure collapse.

What is practical mathematics? Who shall say? The highest theoretical mathematics of fifty years ago is now applied to science in such a way as to affect our industries. Who shall be responsible for asserting that the highly abstract branches of mathematics of today shall never find application in civilized society? Mathematics is one of the long range eyes of science and industry. Mathematical theories of physical phenomena have pointed the way to great inventions long before the so-called practical man realized their possibility. Mathematics has truly a prophetic eye. While there is not time for more than general statements here, there is not lacking all desirable historical evidence and specific cases to support the claims made. The instances are not exceptional but the general rule in scientific and industrial development.

We hear much about the "practical" in elementary and secondary mathematics. Even the college teacher is beset with the query "what is this or that piece of mathematical work going to net me in my profession?" You will possibly be surprised that students of engineering are the most frequent in such inquiries. Strange it is, too, when without mathematics the profession of engineering could not exist. The

advocates of practical mathematics range all the way from the pure empiricist, who cares only for a few formulæ used in the routine of his little world of business or profession, to the "projectionist" who must have a "project," real or imaginary, for every new mathematical step in the whole course. The first are evidently too narrow to perpetuate even their own interests and could not reproduce their formulæ if lost nor can they intelligently extend their mathematics beyond their present attainments. They cry against what has made them possible and what will keep them going. The last are so absorbed in their projects that there is danger that they may overlook the true spirit of mathematics and submerge it in a mist of material interests. There must be a reasonable mixture of the theory and its material connections. But to reduce the theory to a mere incident in the development of a material interest will defeat the purpose of the materialist himself.

Our educational system must always furnish a surplus of possible investigators in pure mathematics by giving a considerable number of pupils an opportunity to try out the field of mathematics. What is said of mathematics may be said of other instrumental branches. Whenever the world catches up, so to speak, with mathematical research, by having made use, in science or industry, of all available mathematical knowledge (and this would not be impossible if some educators had their way), civilization must come to a halt until further researches are made. Any nation that permits its educational system to omit or neglect a subject so fundamental and universal will suffer in the next generation. Modifications and adaptations to meet new situations we must admit and encourage; elimination, never. We as a professional group will be heard in proportion to our capacity and readiness to meet the actual problems of civilization successfully.

In order to facilitate the constantly present need of adjustments in our subject, it is highly desirable that teachers of mathematics have some side line whereby they may keep in contact with affairs. This will be a source of support and encouragement and a reservoir from which to draw live problems to illustrate and enforce the teaching.

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